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Statistical Approach to Analyze Vibration Localization Phenomena in Periodic Structural Systems

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Malfunctions or critical fatigue problems often occur in mistuned periodic structural systems since their vibration responses may become much larger than those of perfectly tuned periodic systems. These are called vibration localization phenomena and it is of great importance to accurately predict the localization phenomena for safe and reliable designs of the periodic structural systems. In this study, a simple discrete system which represents periodic structural systems is employed to analyze the vibration localization phenomena. The statistical effects of mistuning, stiffness coupling, and damping on the vibration localization phenomena are investigated through Monte Carlo simulation. It is found that the probability of vibration localization was significantly influenced by the statistical properties except the standard deviation of coupling stiffness.

Key Words: Vibration Localization, Periodic Structures, Largest Frequency Response Amplitude, Statistics, Probability, Monte Carlo Simulation

1. Introduction

Periodic structural systems, in which identical subcomponents are repeated, can be found in several engineering examples such as turbine and helicopter blades. Periodic structures, however, are not perfectly periodic since there always exist manufacturing tolerances, disorders of material properties, and operational wear in their subcomponents. Such a structural system which has at least one slightly different subcomponent is called a mistuned (or disordered) periodic system. The forced vibration response of some subcomponent of a mistuned periodic system becomes often much larger than that of a perfectly tuned periodic system. Thus, the vibration energy tends to concentrate on a few subcomponents of the mistuned periodic system. These phenomena, which are called the vibration localization phenomena, often cause unexpected premature failures in the periodic structural systems. Therefore, the effect of mistuning on the vibration localization phenomena has been investigated by several researchers.

Since the pioneering work of Anderson (1958) on localization in disordered periodic systems in solid state physics, the localization phenomena have drawn attention from many researchers in mechanical engineering. Ewins (1969, 1970, 1973, 1976) showed that the maximal forced response

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increases with increasing mistuning up to certain level. However, further increase of mistuning results in lower forced response. Hodges (1982) was the first one to recognize the relevance of localization theory to dynamical behavior of periodic structures in structural dynamics. Bendiksen (1984, 1987) investigated mode localization of mistuned turbo-machinery rotors using disordered chain of coupled pendulum. Pierre et al. (1987a, 1987b) investigated mode localization of disordered multi-span beams and explained mode localization by perturbation method using disordered chain of coupled pendulum. Wei et al. (1988), Castanier et al. (1997) and Choi (2003) also introduced intentional mistuning into the design of bladed disks in order to reduce the maximal forced response. Recently, Yoo et al. (2003) investigated the localization phenomena by analyzing the forced vibration responses. They found a certain relation between mistuning and stiffness coupling which causes strong localization. The effect of damping on the localization was also investigated. In the literature so far mentioned, however, the system parameters were assumed to possess deterministic properties rather than statistical properties. Only a few authors (Sogliero, 1980; Sinha, 1986) have acknowledged the need to employ statistical approaches to investigate the localization phenomena so far.

In this paper, the statistical effects of mistuning, stiffness coupling, and damping on the vibration localization phenomena are investigated. A simple coupled pendulum is employed to investigate the localization phenomena in mistuned periodic systems. Monte Carlo simulation method (Rubinstein, 1981), which is a powerful tool based on the theory of probability (Papoulis, 1984), is employed to obtain the numerical results.

2. Simplified Modeling for Coupled Periodic Structural Systems

Periodic structural systems have repeated subcomponents that have identical structural topology including geometry, coupling stiffness, and damping. The mistuning of a periodic structural system results from irregularities such as length difference. Figure 1 shows a planar coupled pendulum system. Each pendulum has torsional spring of modulus k_r and two translational springs of modulus k_t. The distance from hinge point to translational spring is a. The notation d_i denotes a length difference which represents the mistuning of the i-th pendulum. Even though damping symbols do not appear in Fig. 1, linear viscous proportional damping force (with damping constant c) is assumed to act on each pendulum mass. If each pendulum mass is excited by random excitation force f^i, the equations of motion of the i-th pendulum are derived as follows:

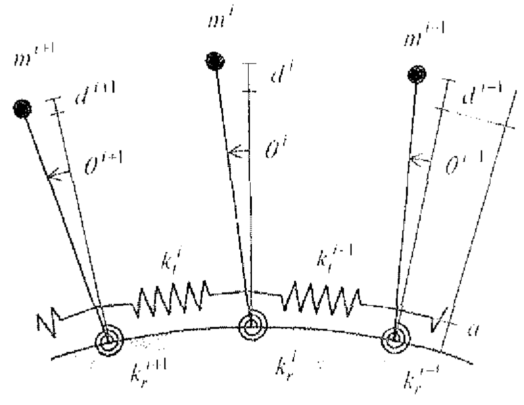


Fig. 1 Multiple coupled pendulum system

$$\begin{aligned} &m^i(l+d^i)^2\ddot{\theta}^i+c^i(l+d^i)^2\dot{\theta}^i+k_r^i\theta^i \\ &-k_t^{i-1}a^2\theta^{i-1}+(k_t^{i-1}+k_t^i)a^2\theta^i \\ &-k_t^i a^2\theta^{i+1}=f^i(l+d^i) \qquad (i=1,\ 2,\ \cdots,\ n) \end{aligned} \tag{1}$$

where l represents a nominal length of pendulums. Assuming the same properties of mass m, damping constant c, torsional spring modulus k_r, translational spring modulus k_t, and external force f, Eq. (1) can be written as follows:

$$\begin{aligned} &m(l+d^i)^2\ddot{\theta}^i+c(l+d^i)^2\dot{\theta}^i+k_r\theta^i \\ &-k_t a^2\theta^{i-1}+2k_t a^2\theta^i-k_t a^2\theta^{i+1}=f(l+d^i) \\ &\qquad (i=1,\ 2,\ \cdots,\ n) \end{aligned} \tag{2}$$

To obtain more general and useful conclusions from the equations of motion, dimensionless parameters and a dimensionless variable are defined as follows:

$$a_i \equiv \frac{d^i}{l},\ \beta \equiv \frac{k_t a^2}{k_r},\ \zeta \equiv \frac{cl}{2\sqrt{k_r m}}$$
$$\gamma \equiv \frac{fl}{k_r},\ \tau \equiv \sqrt{\frac{k_r}{ml^2}}\,t \tag{3}$$

where a_i, β, ζ, γ, and τ represent length mistuning, coupling stiffness between subcomponents, damping, external force, and a dimensionless time, respectively. Employing these dimensionless parameters and variable, Eq. (2) can be rewritten as

$$\begin{aligned}&(1+a_i)^2\ddot{\theta}^i + 2\zeta(1+a_i)^2\dot{\theta}^i - \beta\theta^{i-1}\\ &+(1+2\beta)\,\theta^i - \beta\theta^{i+1} = \gamma(1+a_i)\\ &\qquad (i=1,\ 2,\ \cdots,\ n)\end{aligned} \tag{4}$$

where a dot over a symbol now represents the differentiation of the symbol with respect to dimensionless time variable τ. Therefore, the equations of motion can be written as follows:

$$[M]\{\ddot{\theta}\} + 2\zeta[M]\{\dot{\theta}\} + [K]\{\theta\} = \gamma\{F\} \tag{5}$$

where

$$[M]=\begin{bmatrix}(1+a_1)^2 & 0 & \cdots & \cdots & 0\\ 0 & \ddots & \ddots & \ddots & \vdots\\ \vdots & \ddots & (1+a_i)^2 & \ddots & \vdots\\ \vdots & \ddots & \ddots & \ddots & 0\\ 0 & \cdots & \cdots & 0 & (1+a_n)^2\end{bmatrix}\quad \{\theta\}=\begin{Bmatrix}\theta^1\\ \vdots\\ \theta^i\\ \vdots\\ \theta^n\end{Bmatrix}$$
$$[K]=\begin{bmatrix}1+2\beta & -\beta & 0 & 0 & -\beta\\ -\beta & \ddots & \ddots & 0 & 0\\ 0 & -\beta & 1+2\beta & -\beta & 0\\ 0 & 0 & \ddots & \ddots & -\beta\\ -\beta & 0 & 0 & -\beta & 1+2\beta\end{bmatrix}\quad \{F\}=\begin{Bmatrix}(1+a_1)\\ \vdots\\ (1+a_i)\\ \vdots\\ (1+a_n)\end{Bmatrix} \tag{6}$$

Taking Fourier transformation of Eq. (5) yields the following matrix equation.

$$(-\omega^2[M] + j2\zeta\omega[M] + [K])\{\bar{\theta}\} = \bar{\gamma}\{F\} \tag{7}$$

where $\{\bar{\theta}\}$ and $\bar{\gamma}$ are the Fourier transformations of $\{\theta\}$ and γ, respectively. Now, from Eq. (7), one can obtain $\{\bar{\theta}\}$ which represents the frequency response of the pendulum system. The frequency response amplitude of the i-th pendulum is $|\bar{\theta}_i|$ (whose maximum will be denoted as κ_i in the next section). Substituting $\bar{\theta}_i = X_i + jY_i$ into Eq. (7), the following equation can be derived.

$$\begin{bmatrix}-\omega^2[M]+[K] & -2\zeta\omega[M]\\ 2\zeta\omega[M] & -\omega^2[M]+[K]\end{bmatrix}\begin{Bmatrix}\{X\}\\ \{Y\}\end{Bmatrix} = \bar{\gamma}\begin{Bmatrix}\{F\}\\ 0\end{Bmatrix} \tag{8}$$

The excitation force is assumed to have white random property in this study. So, a constant value for $\bar{\gamma}$ is employed to obtain X_i and Y_i from Eq. (8). However, $\bar{\gamma}$ may be given differently if it is known to the analyzer.

In the next section, the vibration localization phenomena occurred in the mistuned pendulum system will be investigated by employing the Monte Carlo simulation method. The input random parameters in this method are the mistuning parameter a_i, the coupling stiffness parameter β and the damping parameter ζ. The output random parameter is the largest frequency response amplitude κ_i, that is the maximum of $|\bar{\theta}_i|$. The Monte Carlo simulation can be described as follows. Normally distributed patterns for length disorder, coupling stiffness, and damping are obtained by using a random number generator. A set of mistuned systems determined by the patterns is analyzed through the equations mentioned above. The largest frequency response amplitude constitutes one sample and is stored in a vector. This process is repeated until some convergence condition is met. Post-processing of the sample vector results in an approximation of the response statistics. The effects of the statistical properties of the three dimensionless parameters on the localization phenomena will be investigated through this procedure.

3. Numerical Results and Discussion

The simplest case of the coupled pendulum system, that is the case of $n=2$, is investigated in this study. First, let's consider cases in which all the system parameters (a_i's, β, and ζ) are assumed to be deterministic rather than statistical. To obtain the numerical results, the following three sets (for a_1, a_2, and β) are employed. The three sets are (0.0, 0.0, 0.002), (0.0, 0.024, 0.002), and (0.0, 0.024, 0.012) and ζ is always equal to 0.005. Especially, the third set is the system parameters at which the maximum response appears. It is identical with that obtained by using the modeling method of Yoo et al.(2003). Numerical results for the three sets of parameters are shown in Fig. 2. Figure 2(a) shows the results of the first case. Two pendulums are identical in this case so that their frequency response curves are identical, too. Figures 2(b) and (c) show the results of the second and the third cases. As shown in the figures, the length mistuning (the difference between α_1 and α_2) causes the differences of frequency response curves. Comparing the two results, one can realize that the weak coupling stiffness (β=0.002) results in smaller difference. Therefore, the first parameter set (0.0, 0.0, 0.002) can be employed to represent a tuned system while the third set (0.0, 0.024, 0.012) can be employed to represent a mistuned system. The highest peak of the frequency response curve of the mistuned system divided by that of the tuned system, κ_2/κ_{tuned}, is employed to describe the vibration localization criterion in this study. For instance, one may employ $\kappa_2/\kappa_{tuned} > 1.1$ as a vibration localization criterion. In other words, vibration localization presumes to occur if κ_2/κ_{tuned} is greater than 1.1.

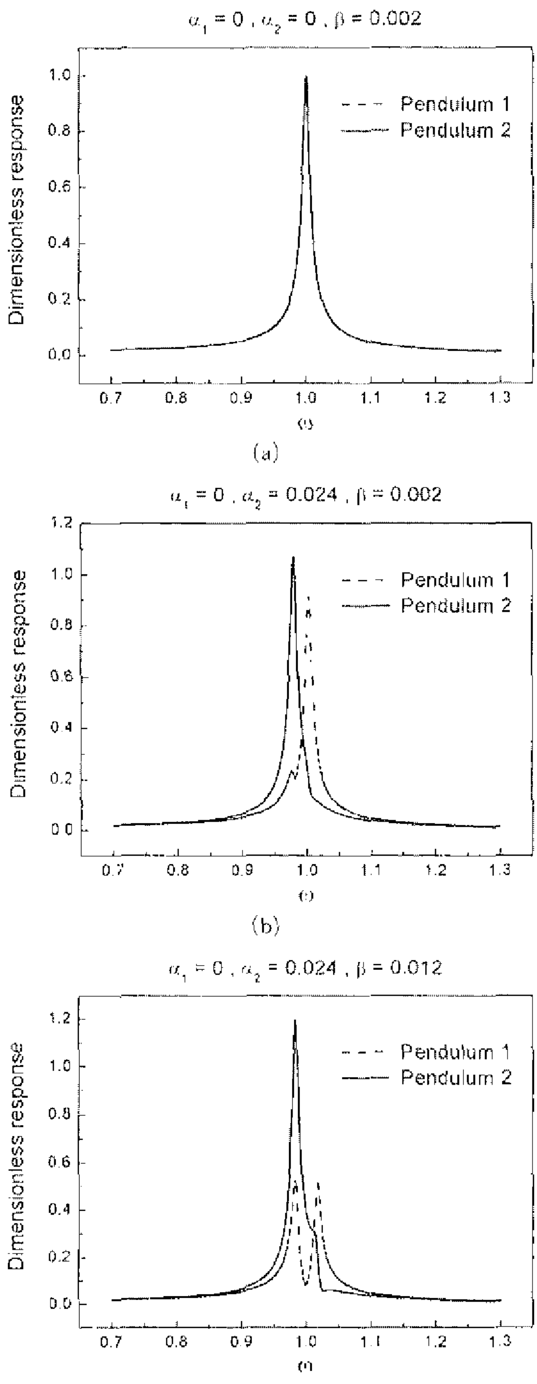


Fig. 2 Frequency response curves of the three deterministic cases

To investigate the effects of the statistical properties of the length mistuning parameters (α_i's) on the vibration localization phenomena, Monte Carlo simulation was performed. First, to verify the validity of Monte Carlo simulation, the convergence test was made. Table 1 shows the convergence trends of the mean and the standard deviation of κ_2/κ_{tuned}. To obtain the results in Table 1, $E(\alpha_1)=0$, $E(\alpha_2)=0.024$ and $\beta=0.002$ are employed for the tuned case and $E(\alpha_1)=0$, $E(\alpha_2)=0.024$ and $\beta=0.012$ are employed for the mistuned case. For the two cases, the standard deviations (of α_i's) are 0.01 and ζ is equal to 0.005. The probability (that κ_2/κ_{tuned} is greater than 1.1) is also given in Table 1. It is shown that the simulation results converge rapidly as the number of sampling data increases. So one may conclude that the Monte Carlo simulations provide reasonable convergent numerical results as the number of sampling data increases. To save computation time while maintaining simulation accuracy, hereinafter, ten thousand sampling data are employed to obtain the numerical results.

Figure 3 shows the probability (that κ_2/κ_{tuned} is greater than 1.1 and 1.05) versus the standard deviations of α_1 and α_2 for the tuned case. The darker areas indicate relatively higher probability. Even if this system is a tuned system (that is designed intentionally to avoid vibration localization), relatively higher probability of vibration localization still exists in some specific ranges of standard deviations of α_1 and α_2. As can be observed from Fig. 3(a), there exists a quarter circle band in which the probability of vibration localization is relatively higher than the rest area. So one may reach a useful conclusion for practical designs. To avoid higher probability of vibration localization (that κ_2/κ_{tuned} is greater than 1.1), the root of the square sum of the standard deviations should avoid the range that is approximately $0.005 \leq \sqrt{\sigma^2_{\alpha_1}+\sigma^2_{\alpha_2}} \leq 0.012$. Figure 3 (b) also shows that the range of relatively higher probability expands significantly as the localization criterion is lowered from $\kappa_2/\kappa_{tuned}<1.1$ to $\kappa_2/\kappa_{tuned}<1.05$. In this case, to avoid the vibration localization (that $\kappa 2/\kappa_{tuned}$ is greater than 1.05), one should significantly reduce the standard deviations of α_1 and α_2 which are directly related to the manufacturing tolerances. Of couse, that will naturally increase the manufacturing cost.

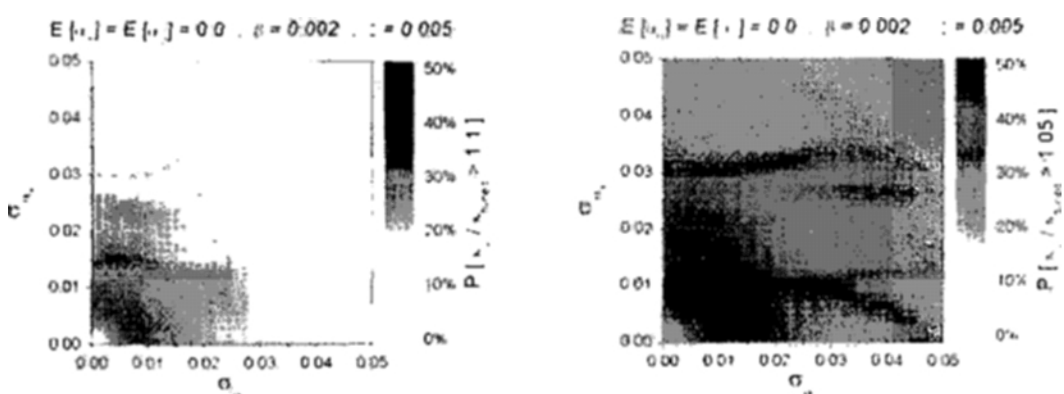
Figure 4 shows the results of the mistuned case in which $E(\alpha_1)=0$, $E(\alpha_2)=0.024$ and $\beta=0.012$. Different from the previous results of the tuned case, relatively higher probability range is concentrated near the origin. To ensure the higher probability of the vibration localization (that κ_2/κ_{tuned} is greater than 1.1), the root of the square sum of the standard deviations should be approximately less than 0.015. Figure 4(b) also shows that the range of high probability only slightly expands as the localization criterion is lowered from 1.1 to 1.05.

Figure 5 shows the probability of κ_2/κ_{tuned} (that is greater than 1.1 and 1.05) versus the same standard deviation of α_i's and the mean of α_2

Table 1 Convergence of the response statistics as the number of sampling data increases

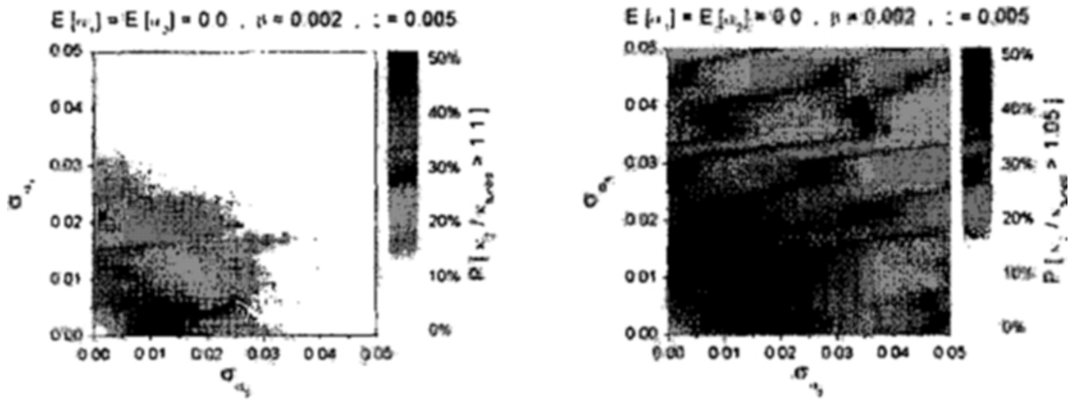
Number of Samples	Mean		Standard deviation		Probability	
	tuned	mistuned	tuned	mistuned	tuned	mistuned
10	0.9094	1.1794	0.1266	0.0155	0.2000	1.0000
100	0.9820	1.1616	0.1404	0.0527	0.3200	0.9000
1000	0.9699	1.1546	0.1335	0.0825	0.2740	0.8980
10000	0.9746	1.1573	0.1314	0.0745	0.2737	0.9008
100000	0.9767	1.1575	0.1316	0.0728	0.2786	0.8989

*tuned case : $E(\alpha_1)=0$, $E(\alpha_2)=0$, $\sigma_{\alpha_i}=0.01$, $\beta=0.002$, $\zeta=0.005$
*mistuned case : $E(\alpha_1)=0$, $E(\alpha_2)=0.024$, $\sigma_{\alpha_i}=0.01$, $\beta=0.012$, $\zeta=0.005$



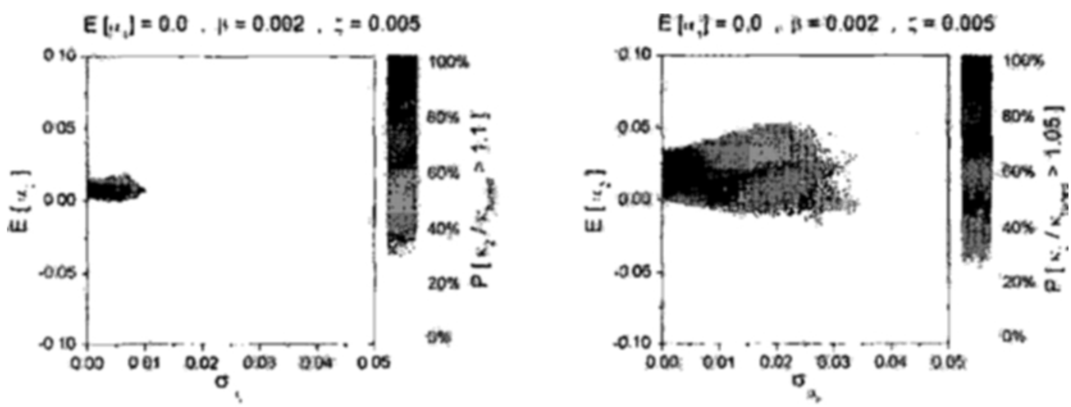
(a) Case of P[$\kappa_2/\kappa_{tuned}>1.1$] (b) Case of P[$\kappa_2/\kappa_{tuned}>1.05$]

Fig. 3 Probability of $\kappa_2/\kappa_{tuned}>$ that is larger than 1.1 and 1.05 versus the standard deviations of α_1 and α_2 for the tuned case



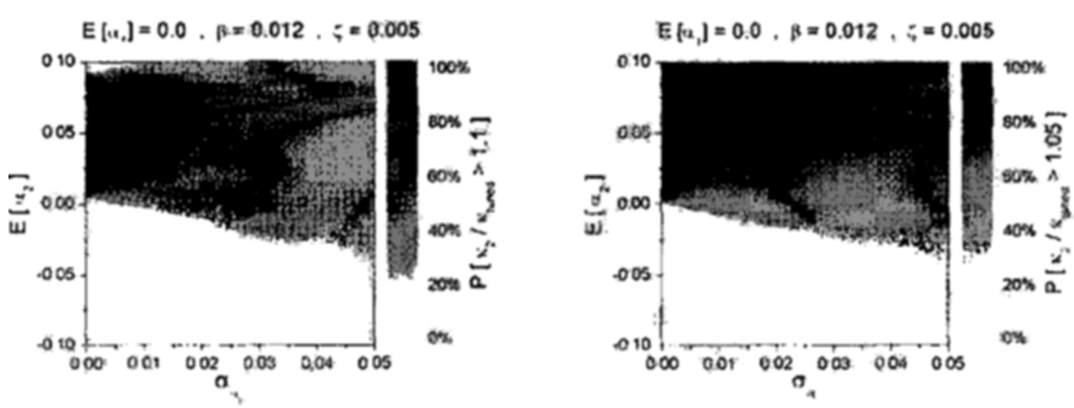
(a) Case of P[κ_2/κ_{tuned}>1.1] (b) Case of P[κ_2/κ_{tuned}>1.05]

Fig. 4 Probability of κ_2/κ_{tuned} that is larger than 1.1 and 1.05 versus the standard deviations of α_1 and α_2 for the mistuned case



(a) Case of P[κ_2/κ_{tuned}>1.1] (b) Case of P[κ_2/κ_{tuned}>1.05]

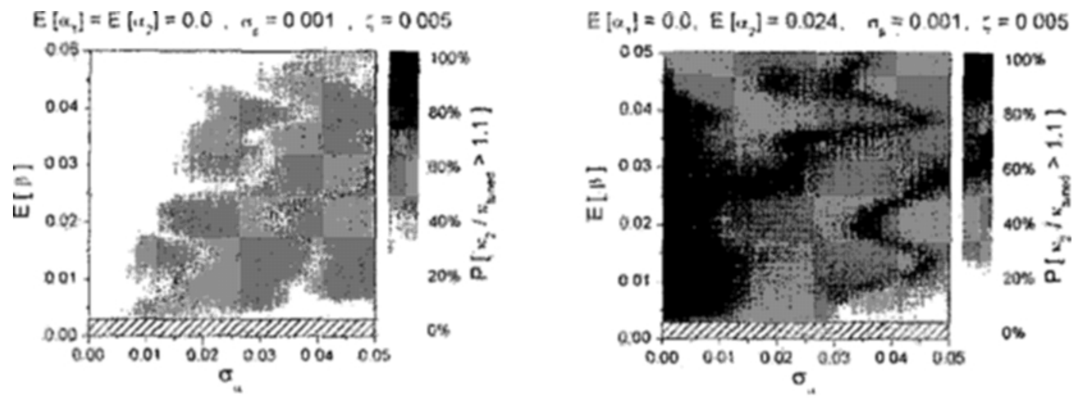
Fig. 5 Probability of κ_2/κ_{tuned} that is larger than 1.1 and 1.05 versus the standard deviation of α_i's and the mean of α_2 for the tuned case



(a) Case of P[κ_2/κ_{tuned}>1.1] (b) Case of P[κ_2/κ_{tuned}>1.05]

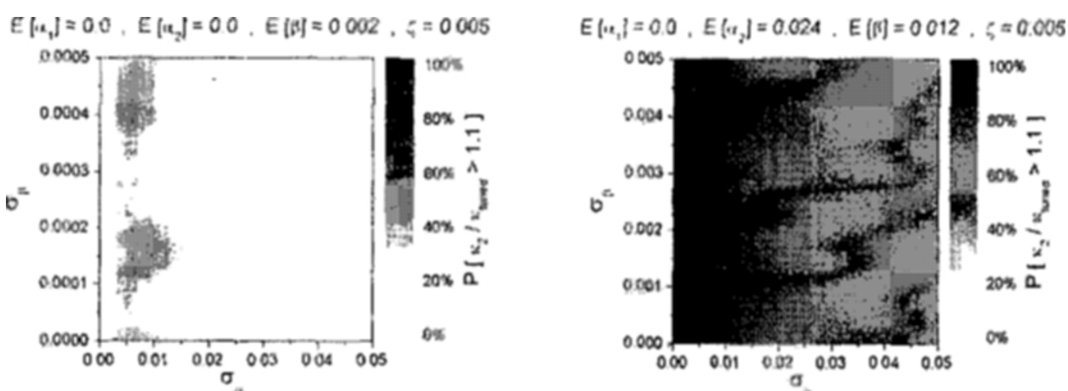
Fig. 6 Probability of κ_2/κ_{tuned} that is larger than 1.1 and 1.05 versus the same standard deviation of α_i's and the mean of α_2 for the mistuned case (when $\beta=0.002$). It can be observed that vibration localization only occurs in a small specific range of standard deviation and mean values. As the criterion is lowered, however, the range expands significantly. Figure 6 shows the probability of κ_2/κ_{tuned} (that is greater than 1.1 and 1.05) versus the same standard deviation of α_i's and the mean of α_2 (when $\beta=0.012$). These results (comparing to those of Figure 5) indicate that the range of vibration localization expands significantly with a specific value of β.

Figure 7 shows the probability of vibration localization (κ_2/κ_{tuned} is greater than 1.1) versus the standard deviation of α_i's and the mean of β for the tuned and mistuned cases. For the tuned case (as shown in Fig. 7(a)), the probability increases as the mean of β increases in proportion to the standard deviation of α_i's. Note that the probability reaches almost 50 percent in the proportional region. Therefore, to avoid the vibration localization, the standard deviation of α_i's and the mean of β should be located inside the upper triangular region near the vertical axis. As the standard deviation of α_i's increases in this region, the mean of β should be also increased to avoid the vibration localization. For the mistuned case (as shown in Fig. 7(b)), the probability of vibration localization (κ_2/κ_{tuned} is greater than 1.1) is almost 100 percent in the region near the vertical axis. The figure also shows that the mean



(a) The tuned case (b) The mistuned case

Fig. 7 Probability of κ_2/κ_{tuned} that is larger than 1.1 versus the standard deviation of α_i's and the mean of β for the tuned and the mistuned cases



(a) The tuned case (b) The mistuned case

Fig. 8 Probability of κ_2/κ_{tuned} that is larger than 1.1 versus the standard deviation of α_i's and the standard deviation of β for the tuned and the mistuned cases of β is desired to be decreased to guarantee the vibration localization. However, if the mean of β is decreased excessively (approximately less than 0.005), the probability decreases rapidly.

When the mean value is less than three times the standard deviation, the probability to generate a positive sample is less than 99.73% and a few negative samples can be generated by random number generator. Since negative coupling stiffness provides meaningless results, the region (in which negative coupling stiffness can be generated) is removed from Fig. 7 with hatching.

Figure 8 shows the probability of vibration localization (κ_2/κ_{tuned} is greater than 1.1) versus the standard deviation of α_i's and the standard deviation of β for the tuned and mistuned cases. The figure shows that the probability is hardly influenced by the standard deviation of β in both cases. The standard deviation of α_i's should not exceed 0.003 to avoid the vibration localization for the tuned case (as shown in Fig. 8(a)) while it should not exceed 0.01 to guarantee the vibration localization for the mistuned case (as shown in Fig. 8(b)). Comparing the two numbers, one can conclude that it is more difficult to avoid the vibration localization than to guarantee the vibration localization.

4. Conclusions

A simple discrete system which represents periodic structural systems was employed to investigate the vibration localization phenomena through a statistical method. The effects of statistical properties of the length mistuning, the coupling stiffness and the damping on the probability of vibration localization were examined through Monte Carlo simulation. It was found that the probability of vibration localization was significantly influenced by the statistical properties except the standard deviation of coupling stiffness. In conclusion, the vibration localization could be either avoided or guaranteed, for the design of periodic structural systems, by controlling the statistical properties of the length mistuning, the coupling stiffness, and the damping.

Acknowledgments

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References

Anderson, P. W., 1958, "Absence of Diffusion in Certain Random Lattices," *Physical Review*, Vol. 109, No. 5, pp. 1492～1505.

Bendiksen, O. O., 1984, "Flutter of Mistuned Turbomachinery Rotors," *Journal of Engineering for Gas Turbines and Power*, Vol. 106, No. 1, pp. 25～33.

Bendiksen, O. O., 1987, "Mode Localization Phenomena in Large Space Structure," *AIAA Journal*, Vol. 25 No. 9, pp. 1241～1248.

Castanier, P. and Pierre, C., 1997, "Consideration on the Benefits of Intentional Blade Mistuning for the Forced Response of Turbomachinery Rotors," *Analysis and Design Issues for Modern Aerospace Vehicles*, Vol. 55, pp. 419～425.

Choi, B. K., 2003, "Pattern Optimization of Intentional Blade Mistuning for the Reduction of the Forced Response Using Genetic Algorithm," *KSME International Journal*, Vol. 17, No. 7, pp. 966～977.

Ewins, D. J., 1969, "The Effect of Detuning upon the Forced Vibrations of Bladed Disks," *Journal of Sound and Vibration*, Vol. 9, No. 1, pp. 65～79.

Ewins, D. J., 1970, "A Study of Resonance Coincidence in Bladed Disks," *Journal of Mechanical Engineering Science*, Vol. 12, pp. 305～312.

Ewins, D. J., 1973, "Vibration Characteristics of Bladed Disc Assemblies," *Journal of Mechanical Engineering Science*, Vol. 15, No. 3, pp. 165～186.

Ewins, D. J., 1976, "Vibration Modes of Mistuned Bladed Disks," *ASME Journal of Engineering for Power*, Vol. 98, No. 7, pp. 349～355.

Hodges, C. H., 1982, "Confinement of Vibration By Structural Irregularity," *Journal of Sound and Vibration*, Vol. 82 No. 3, pp. 411~424.

Papoulis, A., 1984, *Probability, Random Variables, and Stochastic Process*, McGraw-Hill.

Pierre, C. and Dowell, E. H., 1987, "Localization of Vibration by Structural Irregularity," *Journal of Sound and Vibration*, Vol. 114, No. 3, pp. 549~564.

Pierre, C., Tang, D. M. and Dowell, E. H., 1987, "Localized Vibrations of Disordered Multispan Beams: Theory and Experiment," *AIAA Journal*, Vol. 25, No. 9, pp. 1249~1257.

Rubinstein, R. Y., 1981, *Simulation and the Monte Carlo Method*, Jon Wiley and Sons.

Sinha, A., 1986, "Calculating the Statistics of Forced Response of a Mistuned Bladed Disk Assembly," *AIAA Journal*, Vol. 24, No. 11, pp. 1797~1801.

Sogliero, G. and Srinivasan, A. V., 1980, "Fatigue Life Estimates of Mistuned Blades via a Stochastic Approach," *AIAA Journal*, Vol. 18, No. 3, pp. 318~323.

Wei, S. T., Pierre, C., 1988, "Localization Phenomena in Mistuned Assemblies with Cyclic Symmetry [Part I] Free Vibrations, [Part II] Forced Vibrations," *Journal of Vibration, Acoustics, Stress, and Reliability in Design*, Vol. 110, pp. 429~449.

Yoo, H. H., Kim, J. Y. and Inman, D. J., 2003, "Vibration Localization of Simplified Mistuned Cyclic Structures Undertaking External Harmonic Force," *Journal of Sound and Vibration*, Vol. 261, No. 5, pp. 859~870.